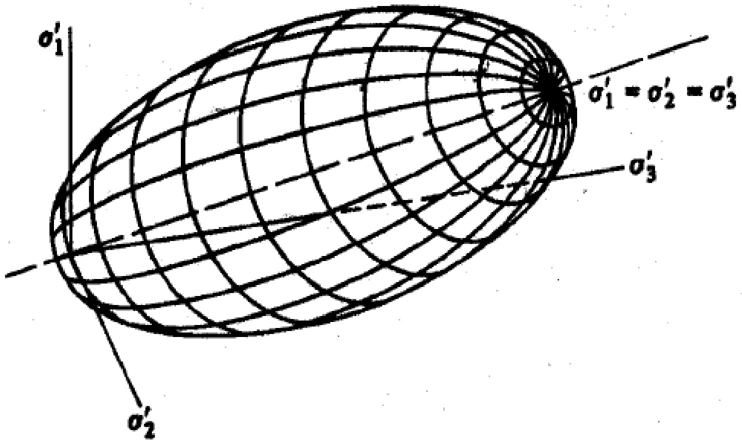

Geomechanics

LECTURE 5

HARDENING ELASTO-PLASTICITY

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Laboratory of soil mechanics – Fall 2024

07.10.2024

Access the QUIZ

https://etc.ch/7UYr

Geomechanics
General Course overview

Semester

Topics

Basic concepts

Classical constitutive framework
- Elasticity - Linear & Non-linear
- Plasticity Perfect-plasticity
- Hardening elasto-plasticity
- Critical state concept
- Modified Cam-Clay

Practical aspects
- Geomechanics in Practice
- Numerical modelling
- Site visit
- In-situ stress
- Course Project
- Retaining structures

Advanced constitutive models
- Unsaturated behavior
- THM behaviour of geomaterials
- Bio-cementation
- Time-dependent behaviour

Content

- Post Yield behavior and plastic strain
- Hardening behaviour
- Consistency condition
- Hardening elasto-plastic stress-strain relationship
- Conclusion

Post Yield behaviour and plastic strain

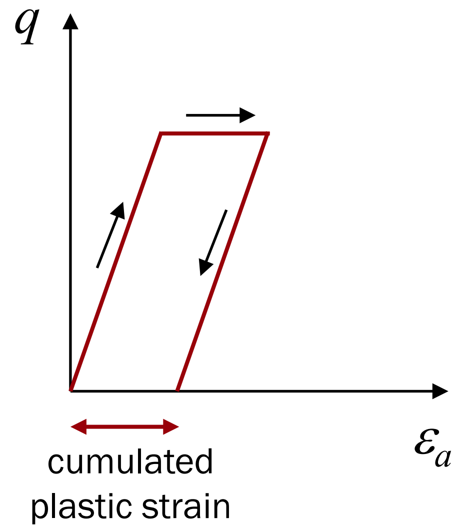
ELASTIC DOMAIN AND YIELD FUNCTION

PLASTIC DEFORMATION

Perfect plasticity vs hardening plasticity

- Perfect plasticity reproduces the accumulation of irreversible deformations, but it assumes yielding = failure.

e.g. elastic-perfectly plastic MC response: occurrence of plastic deformation (**yielding**) when the material reaches the available strength (**failure**).

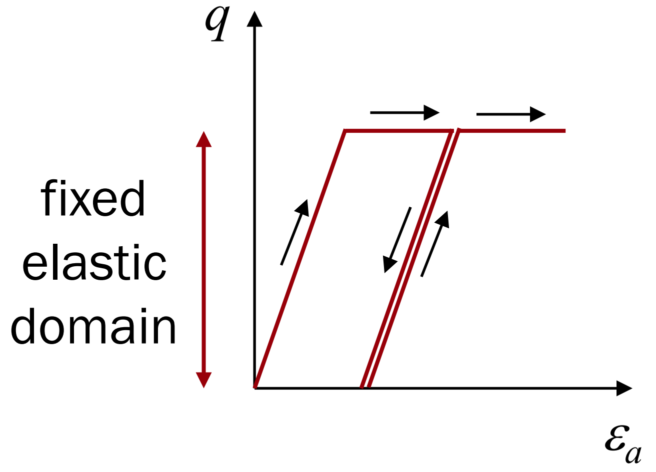


Perfect plasticity vs hardening plasticity

- In perfect plasticity the yield function is fixed, $F = F(\sigma_i, p_k)$

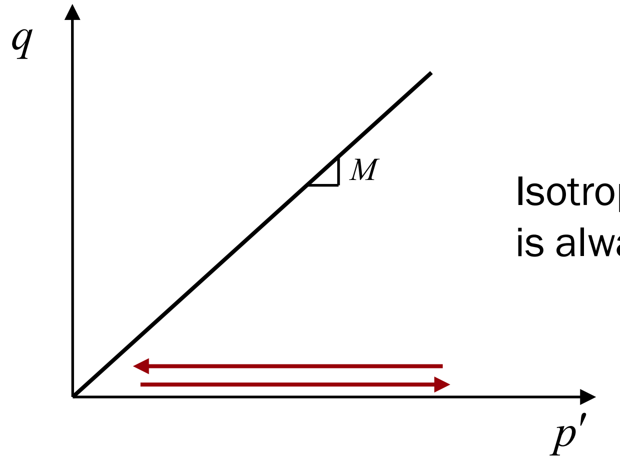
i.e. perfect plasticity doesn't account for the evolution of the elastic domain in shearing.

<ins>Elastic-perfectly plastic MC response</ins>

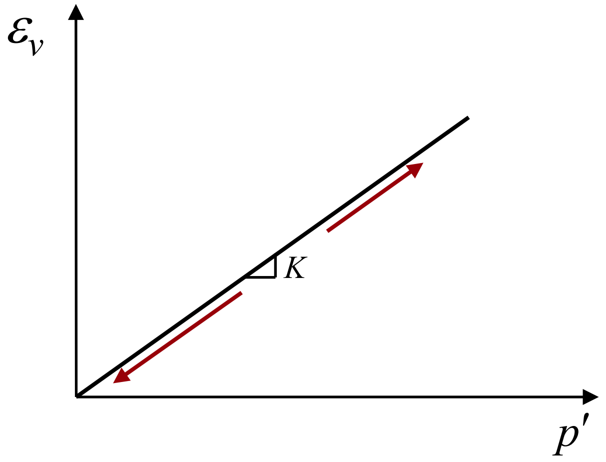


Perfect plasticity vs hardening plasticity

- Other problems of MC elasto-perfectly plastic model:



Isotropic loading/unloading
is always reversible

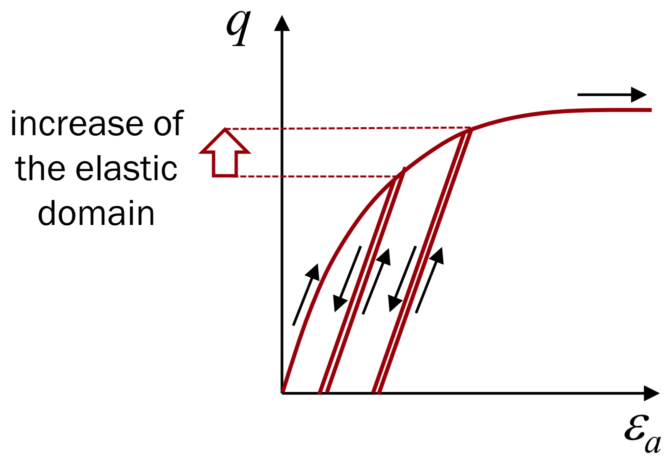


$$\delta\varepsilon_v = \frac{\delta p'}{K}$$

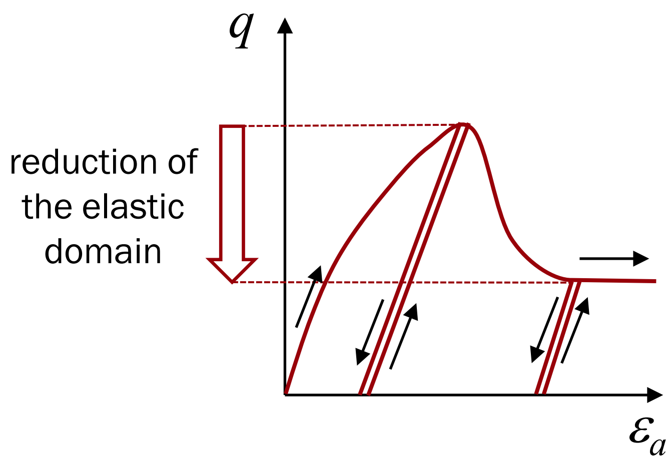
- It is a problem when we are interested in deformation

Perfect plasticity vs hardening plasticity

- The evolution of the elastic domain with loading is a characteristic of geomaterials.



Hardening: expansion of the elastic domain

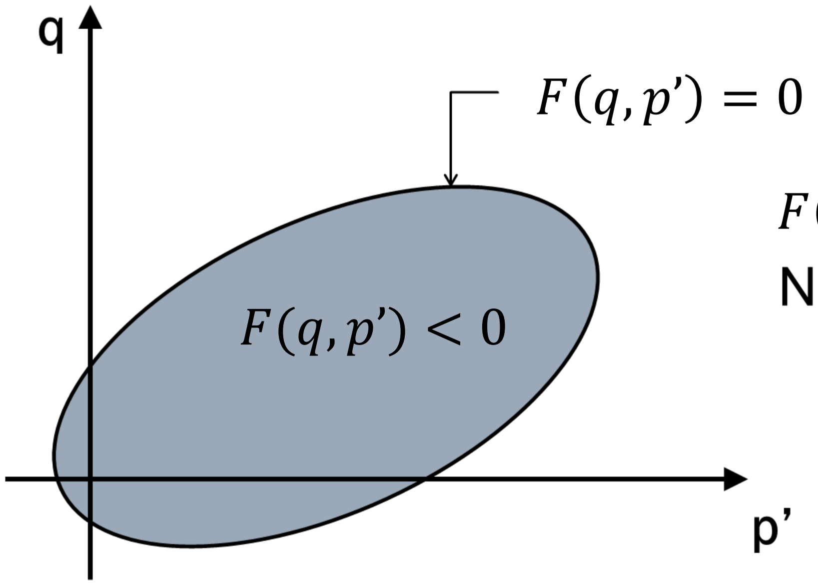


Softening: contraction of the elastic domain

- The difference is related (for the same soil) to OCR (clays) and Relative density (sands).

Elastic domain and yield function

- **Pre-yield behavior**
  Reversible deformation
  Elasticity
- **Yielding**
  Limit between reversible and irreversible behavior
  Yield criteria
- **Post-yield behavior**
  Irreversible deformation
  Full elasto-plastic stress-strain relationship



$F(q,p') > 0$
Not admissible

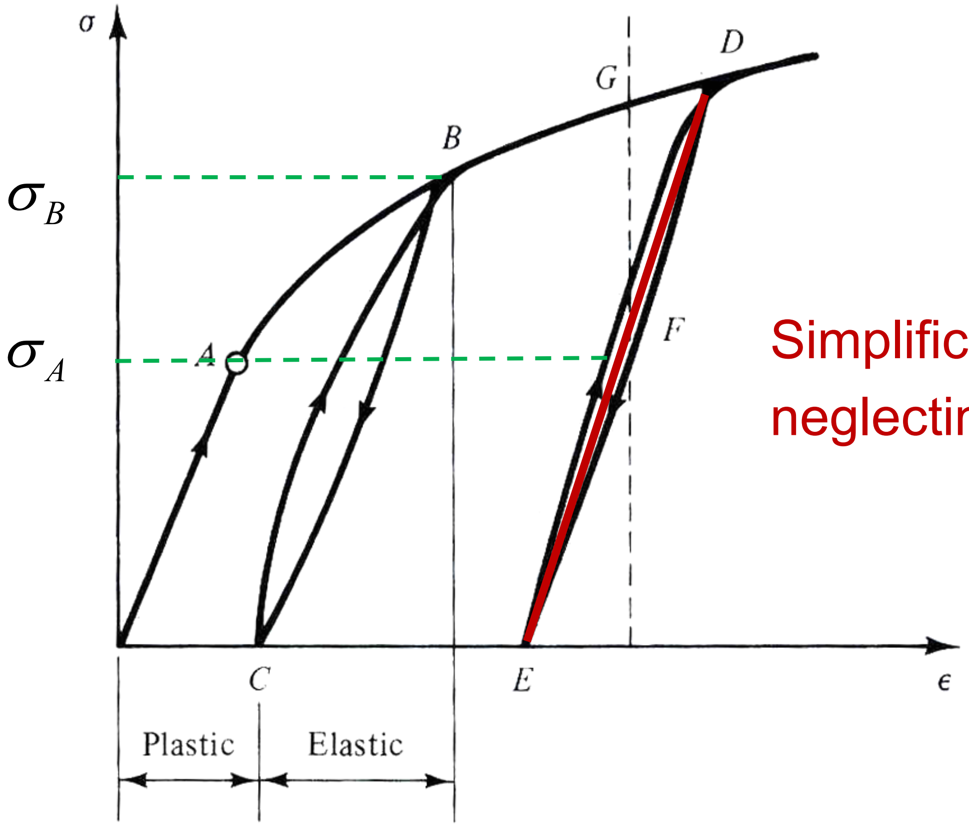
Plastic deformation

Total Elastic Plastic

$$d\varepsilon_{ij} = d\varepsilon_{ij}^{e} + d\varepsilon_{ij}^{p}$$

Strain state depends not only on the actual stress state but also on the stress history

- **“Memory”**



Simplification: neglecting the hysteresis

(Desai & Siriwardane, 1984)

Hardening behaviour

HARDENING BEHAVIOR AND HARDENING RULE

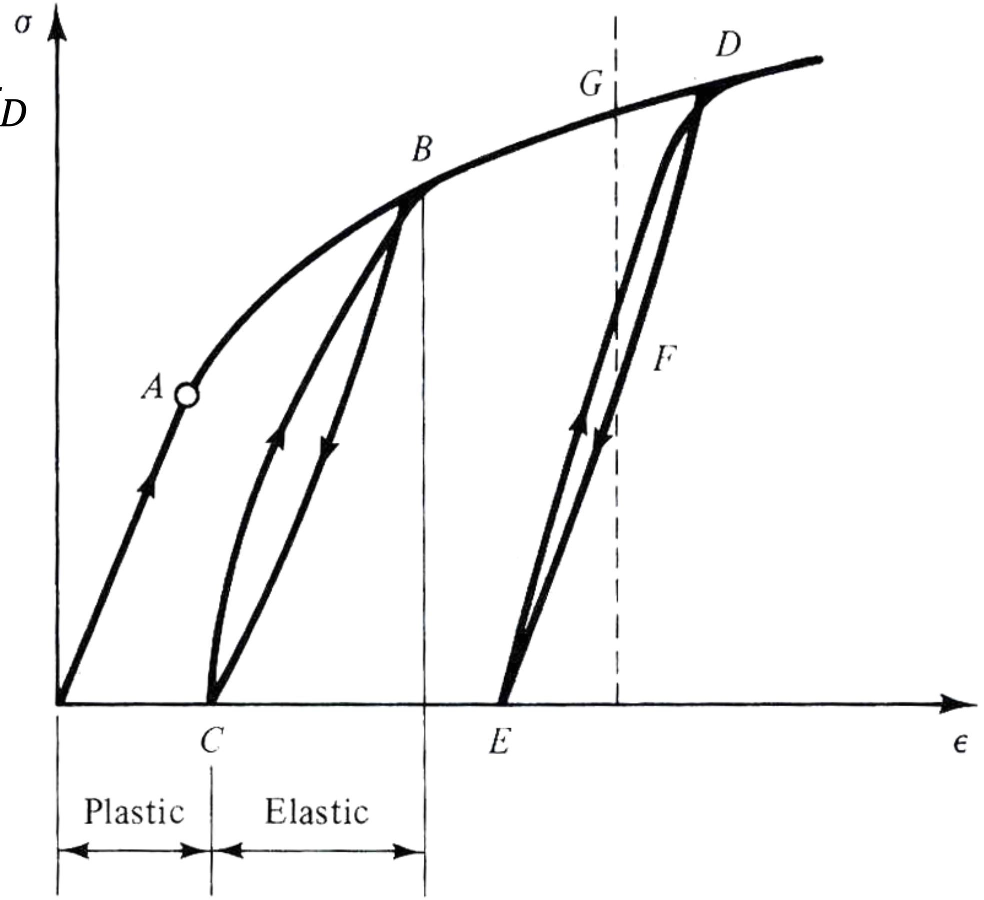
LOADING AND UNLOADING CONDITION

Hardening behaviour

Yield limit evolution

$F_{i+1}: \sigma_D$

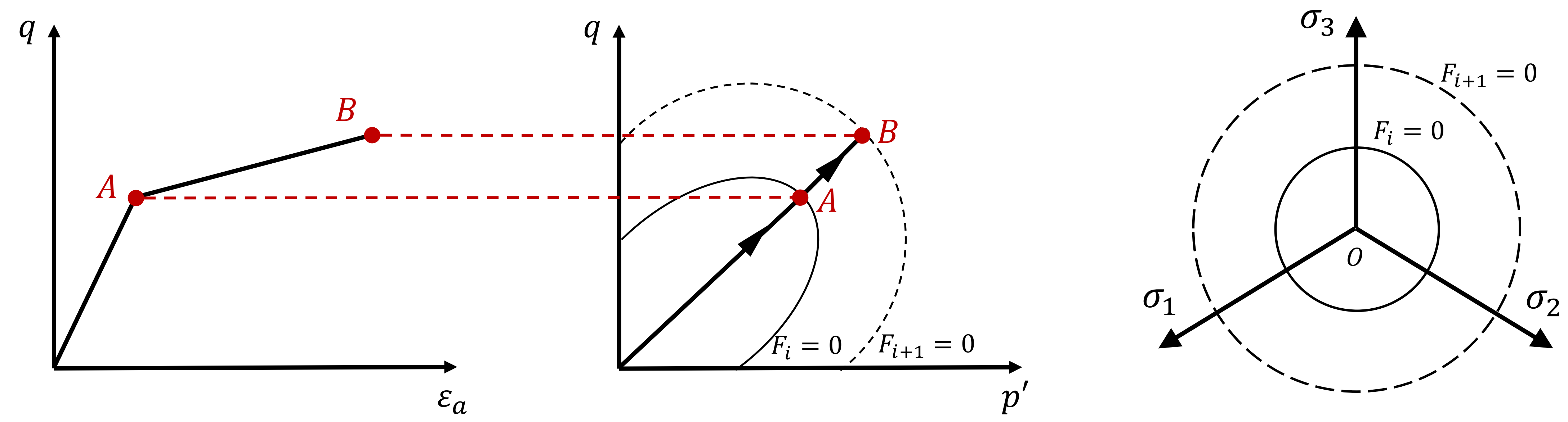
$F_i: \sigma_B$



(Desai & Siriwardane, 1984)

Isotropic hardening

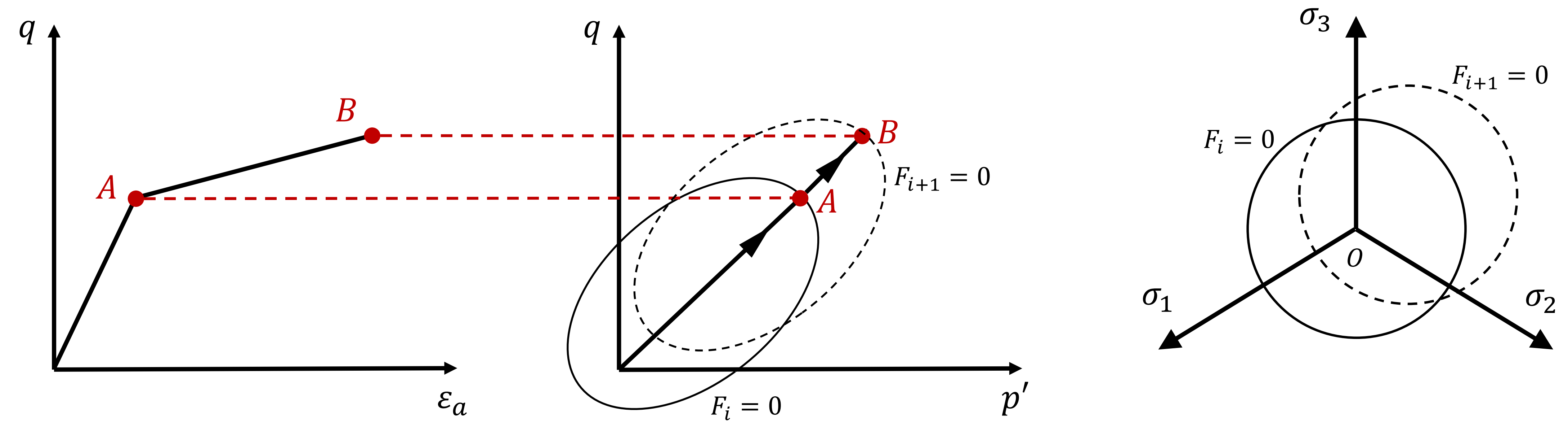
Change in the size of yield limit



Progressive yield surfaces

Kinematic hardening

Change in the position of yield limit



Yield surface remains the same shape and size → rigid transformation

Hardening

- **Yield function**

$$F = F(\sigma_{ij}, p_k) \qquad p_k = \text{geometric parameters}$$

- **Plastic strain hardening**

The evolution of the geometrical parameters is linked to the evolution of plastic strains:

$$p_k = p_k(\varepsilon_{ij}^p) \longrightarrow F = F(\sigma_{ij}, \varepsilon_{ij}^p)$$

Many models use the volumetric plastic strain as hardening (or history) variable (volumetric hardening plasticity)

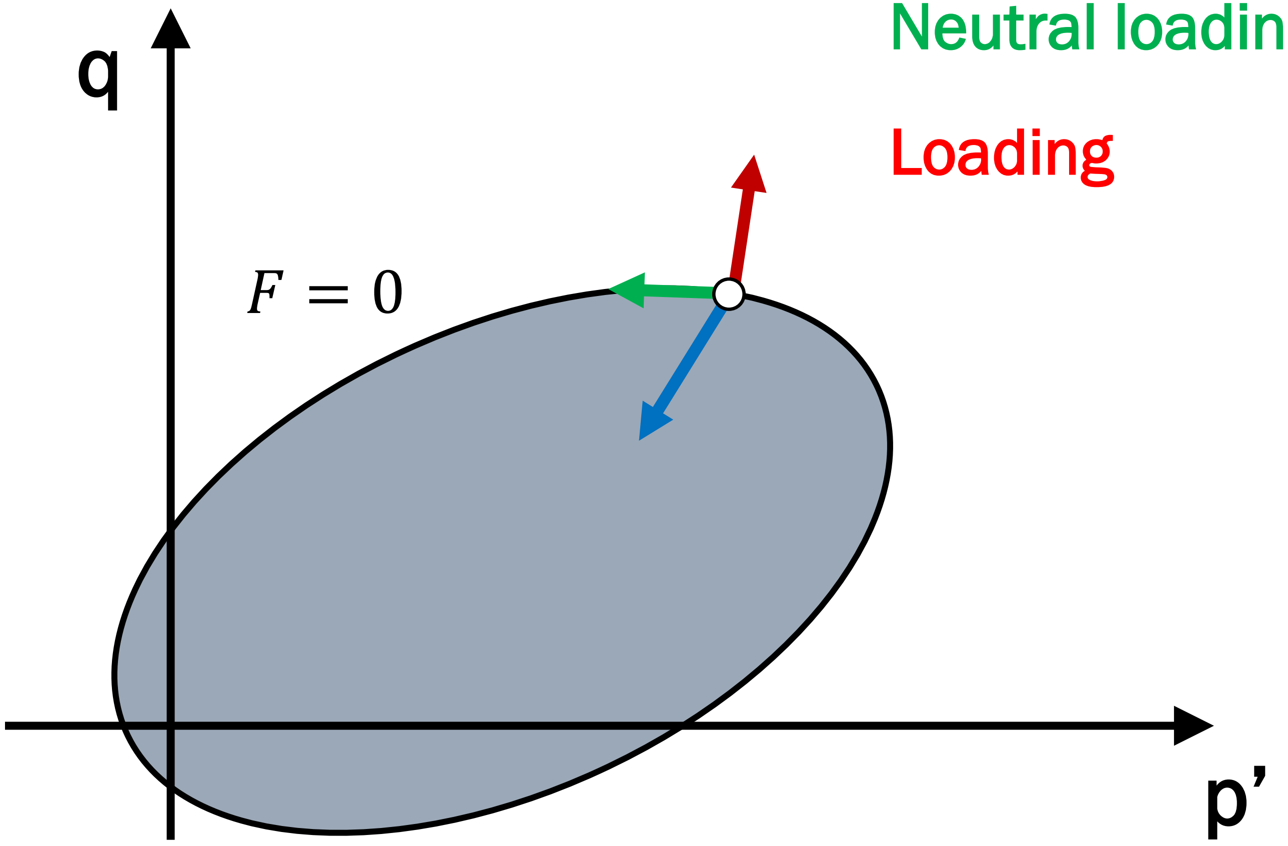
$$F = F(\sigma_{ij}, \varepsilon_v^p)$$

- **Hardening law**

It expresses the link between changes in p_k and changes in plastic strain invariants (i.e. volumetric plastic strains)

$$\delta p_k = function(\varepsilon_{ij}^p)$$

Loading and unloading condition

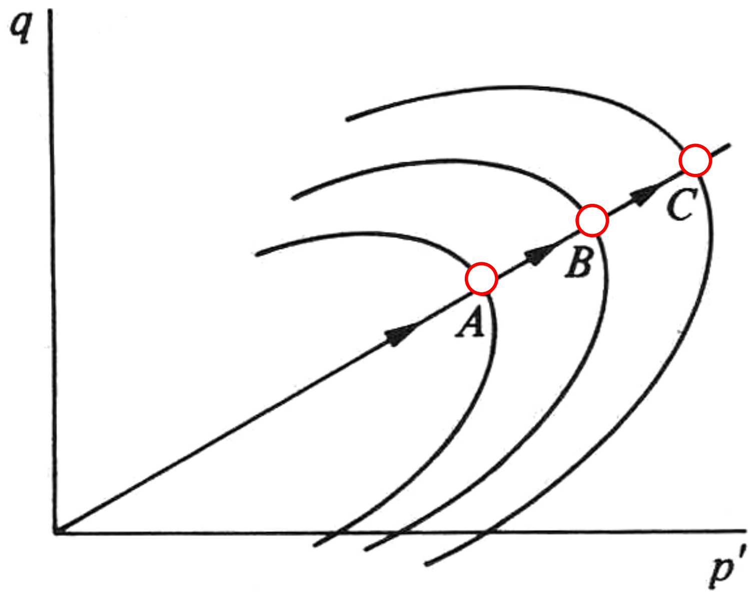


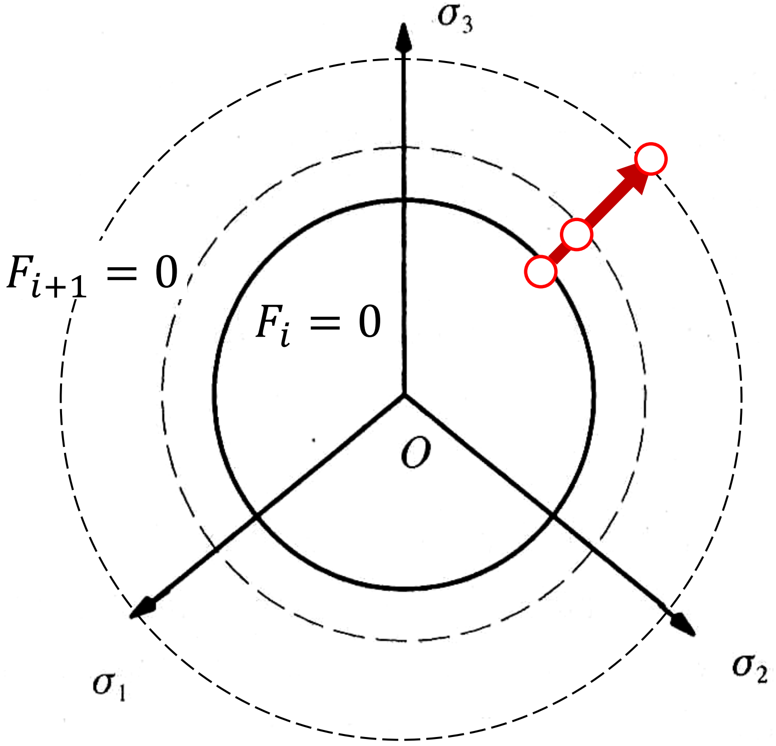
Unloading	$F(\sigma_{ij}, p_k) = 0$	$dF < 0$
Neutral loading	$F(\sigma_{ij}, p_k) = 0$	$dF = 0$
Loading	$F(\sigma_{ij}, p_k) = 0$	$dF > 0$ Plasticity

Consistency conditions

Consistency condition

During plasticity, stress state always stays on the yield surface





Consistency condition

Stress state always on the yield surface

$$dF = \left.\frac{\partial F}{\partial \sigma_i}\right|_{p_k} \delta\sigma_i + \left.\frac{\partial F}{\partial p_k}\right|_{\sigma_i} \delta p_k = 0$$

Plastic strain hardening

$$dF = \left.\frac{\partial F}{\partial \sigma_i}\right|_{p_k} \delta\sigma_i + \left.\frac{\partial F}{\partial p_k}\right|_{\sigma_i} \frac{\partial p_k}{\partial \varepsilon_i^p} \delta\varepsilon_i^p = 0$$

From the hardening rule

Hardening elasto-plastic stress-strain relationship

DERIVATION OF STRESS-STRAIN RELATIONSHIP

Derivation of stress-strain relationship

Elasto-plastic constitutive tensor

Objective: General incremental stress-strain relationship

$$\boxed{\delta\sigma_i = D_{ij}^{ep}\,\delta\varepsilon_j}$$

1. Using elastic constitutive relation $$\delta\sigma_i = D_{ij}^{e}\delta\varepsilon_j^{e} = D_{ij}^{e}\left(\delta\varepsilon_j - \delta\varepsilon_j^{p}\right)$$

2. Replace plastic strain by flow rule $$\delta\sigma_i = D_{ij}^{e}(\delta\varepsilon_j - \mu\frac{\partial g}{\partial\sigma_j})$$

3. Using consistency equation $$dF = \left.\frac{\partial F}{\partial\sigma_i}\right|_{p_k}\delta\sigma_i + \left.\frac{\partial F}{\partial p_k}\right|_{\sigma_i}\frac{\partial p_k}{\partial\varepsilon_i^{p}}\delta\varepsilon_i^{p} = 0$$

4. We obtain plastic multiplier $$\mu = \frac{1}{H}\frac{\partial F}{\partial\sigma_i}\delta\sigma_i \qquad H = -\frac{\delta F}{\delta p_k}\frac{\delta p_k}{\delta\varepsilon_j^{p}}\frac{\delta g}{\delta\sigma_j}$$

Plastic modulus

Derivation of stress-strain relationship

5. Solving for plastic multiplier – using (2) in (4)

$$\mu = \left(\frac{\frac{\partial F}{\partial \sigma_i} D^e_{ij}}{H + \frac{\partial \mathrm{F}}{\partial \sigma_i} D^e_{ij} \frac{\partial g}{\partial \sigma_j}} \right) \delta \varepsilon_j$$

6. Replacing plastic multiplier – (5) in (2)

General incremental stress-strain relationship

Plastic strain isotropic hardening elasto-plasticity

$$\boxed{\delta \sigma_i = \underbrace{\left(D^e_{ij} - \frac{D^e_{ik} \frac{\partial g}{\partial \sigma_l} \frac{\partial F}{\partial \sigma_k} D^e_{kl}}{H + \frac{\partial F}{\partial \sigma_i} D^e_{ij} \frac{\partial g}{\partial \sigma_j}} \right)}_{D^{ep}_{ij}} \delta \varepsilon_j}$$

Conclusion

Conclusion

Ingredients of any elasto-plastic hardening model

(i) **Elastic properties:** to describe the recoverable deformation in elastic domain;

(ii) **Yield function:** to determine the limit when the occurrence plastic deformations starts;

(iii) **Plastic potential**: to understand the mechanism of plastic deformation and direction of plastic strains;

(iv) **Hardening rule:** to determine the magnitude of plastic deformation and evolution of yield surface.

Thank you for your attention